Quantum mechanics. Department of physics, 6th semester.

Lesson №5. ***Mathematical tools of quantum mechanics: momentum representation. One-dimensional (1D)motion: 1D motion of free particle, 1D motion in an infinitely deep square-well potential.***

1. Home task check.

Momentum representation transformation

Tasks 1-2. Find wave functions

$$a).\psi(x) = A\exp\left(ik_0 x\right); \quad b).\psi(x) = A\exp\left(ik_0 x - \frac{x^2}{2a^2}\right)$$

in *p*-representation

$$C(p) = \left(\psi_p, \psi\right) = \int_{-\infty}^{\infty} \psi_p^*(x)\psi(x)dx, \quad \psi_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{ipx}{\hbar}}. \text{ (НКК № 1.42)}$$

Task 3. Find an operator $\hat{p}$ kernel in *p*-representation.

2. Time-dependent Schrodinger equation

$$\boxed{i\hbar\frac{\partial\Psi(q,t)}{\partial t} = \hat{H}\Psi(q,t)},$$

where q means system of generalized coordinates $q_1, q_2, ..., q_n$, t – time.

3. Time-independent Schrodinger equation

$$\frac{\partial\hat{H}}{\partial t} = 0, \quad \Psi(q,t) = \psi(q)e^{-\frac{iEt}{\hbar}},$$

$$\boxed{\hat{H}\psi(q) = E\psi(q).}$$

3.1. Time-independent Schrodinger equation for the particle in constant external field in position representation

$$-\frac{\hbar^2}{2m}\Delta\psi(\vec{r}) + U(\vec{r})\psi(\vec{r}) = E\psi(\vec{r});$$

$\hat{H} = \frac{\hat{\vec{p}}^2}{2m} + U(\hat{\vec{r}}) = -\frac{\hbar^2}{2m}\Delta + U(\vec{r})$ – Hamiltonian of the particle in external field.

3.2. One-dimensional time-independent Schrodinger equation for the particle in constant external field

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{d^2x}+U(x)\psi(x)=E\psi(x).$$

Task 4. Find the general solution of time-dependent Schrodinger equation for one-dimensional free particle. (Hr. № 37).

Task 5. Find energy levels and normalized wave functions of of the particle in an infinitely deep square-well potential at widths $2a$

$$U(x)=\begin{cases}0, & |x|<a,\\ \infty, & |x|>a.\end{cases}$$

4. **Quiz** (~ 20 minutes). Test contains two tasks: 1st task "weights" 10 points, 2nd task "weights" 10 points, so one can get up to 20 points total.

Hometask НКК № 2.1-2.4, 2.7.

НКК- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981

Hr. - Hrechko, Suhakov, Tomasevich, Fedorchenko Collection of theoretical physics problems, 1984